

Short Paper

Formation of Patterns from Complex Networks

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1. Introduction

Geometric patterns appear in nature, and in the art of various cultures. It has been found that some kinds of patterns have a mathematical representation, and grow with a certain parameter like the golden ratio. The patterns have fascinated mathematicians and artists for centuries (ex. Nakayama et al., 2004; Hertzberg and Sweetman, 2005). In network topologies, such patterns appear as well, and they are deeply related to phenomena on networks, for example, flow of people on a transportation network, material flow on a physical network and information flow that occur on a communication network. However, networks in the real world are often too complicated to describe their mathematical structures straightforwardly.

There are many studies about complex network visualization (Schulz and Schumann, 2006; Wiese and Eicher, 2006). Most have focused on presenting hierarchical structures in networks, but not on extracting geometrical patterns from visualized networks. In this paper, many patterns are displayed by visualizing characteristics of network structures, using complex network models and some real-world networks.

2. Layout of Vertices and Edges

Networks are usually represented as graphs $G = (V, E)$, where V and E are sets of vertices and edges respectively. Since graphs show a topological aspect of networks in most cases, their layouts (geometrical positions of vertices) are not easily determined. Furthermore, networks in the real world often have a large-scale structure. Therefore, it is quite difficult to extract a pattern from a network. After several trials, we have noticed that, to utilize a graph layout algorithm based on a specific data structure plays an important role in extracting characteristic patterns. In this paper, we use LGL (Large Graph Layout) to compute the location of vertices. For a given network, LGL calculates a minimum spanning tree (MST), and then arranges the vertices based on the MST (Adai et al., 2004). Edges are laid out according to the adjacency matrix. Visualization method is described below.

3. Patterns from Static Structures

If we visualize a large-scale network by means of drawing vertices and edges straightforwardly, the resultant image becomes too complicated to extract any significant information. In the present method, we do not draw any vertices. Edges are colorized according to a certain quantity.

3.1 Hub Structure

First, we focus on *hubs* (high degree, or highly connected vertices). Edges are colored according to their connectivity; edges connected to low degree vertices are expressed by a cold (bluish) color, and highly connected edges by a warm (reddish) color. In Fig. 1, several theoretical models of complex networks, Barabási-Albert (BA) model (Barabási and Albert, 1999), Klemm-Eguílz (KE) model (Klemm and Eguílz, 2002) and Connecting Nearest Neighbor (CNN) model (Vázquez, 2003) are visualized. Although it is known that similar degree distributions appear in all the models, spatial structures of hubs are difficult to be observed. Our visualization reveals different characteristic patterns of hub structure.

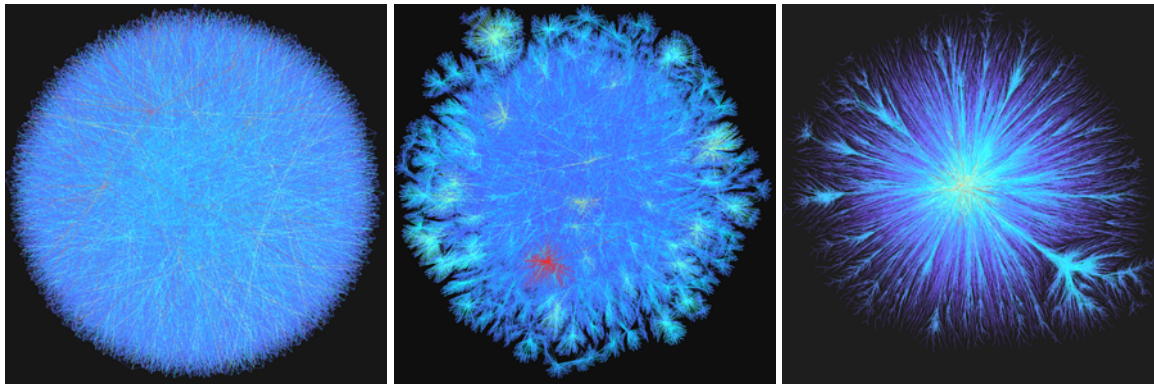


Fig. 1. Visualization of static structures (left: BA model, middle: KE model, right: CNN model).

3.2 Community Structure

Secondly, we remark *communities*, which are defined as a set of cohesive subgraphs. The edges that are components of communities are visualized by different colors. Figure 2 shows the communities of a Weblog network classified by Newman's method (Newman and Girvan, 2004). It is found that the colorization of the cohesive communities is effective to extract a unique pattern of network structure.

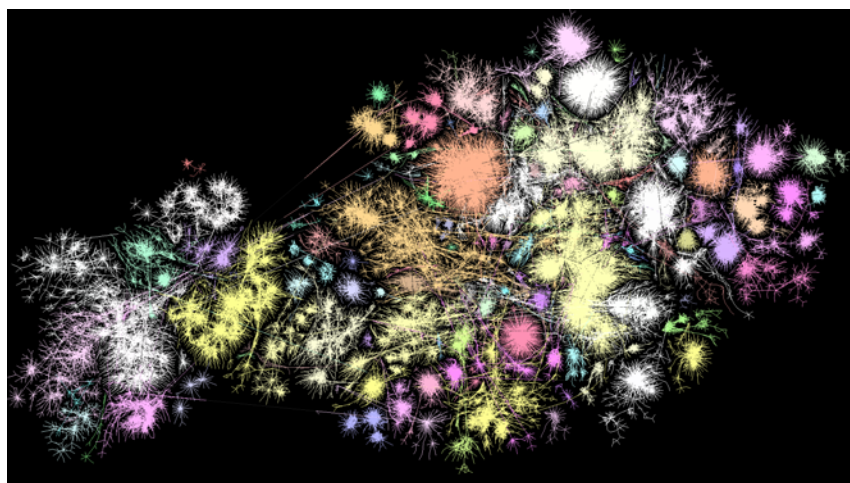


Fig. 2. Visualization of community structure (Weblog network).

4. Growing Patterns of Networks

Networks often have a feature of growing. In several network models, a growing process is modeled as well. In order to visualize a process of network growing, we colorize edges according to their age. First, vertices at final stage are laid out. Secondly, the edges generated at earlier stage of the evolution are represented by a warm color, and then new edges colored by a cold color are drawn. Several characteristic patterns in the process of network growing are shown in Fig. 3.

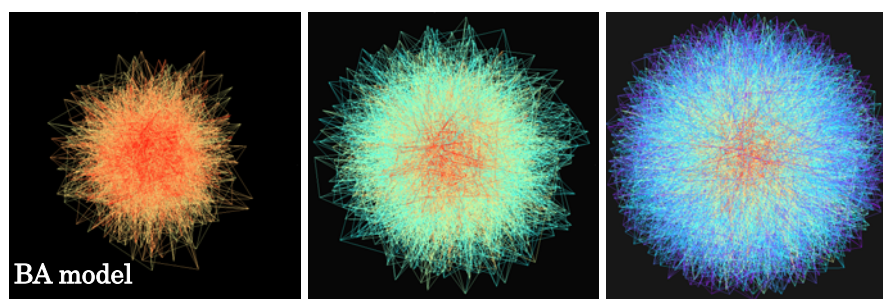


Fig. 3. Visualization of growing patterns (in three different evolving stages).

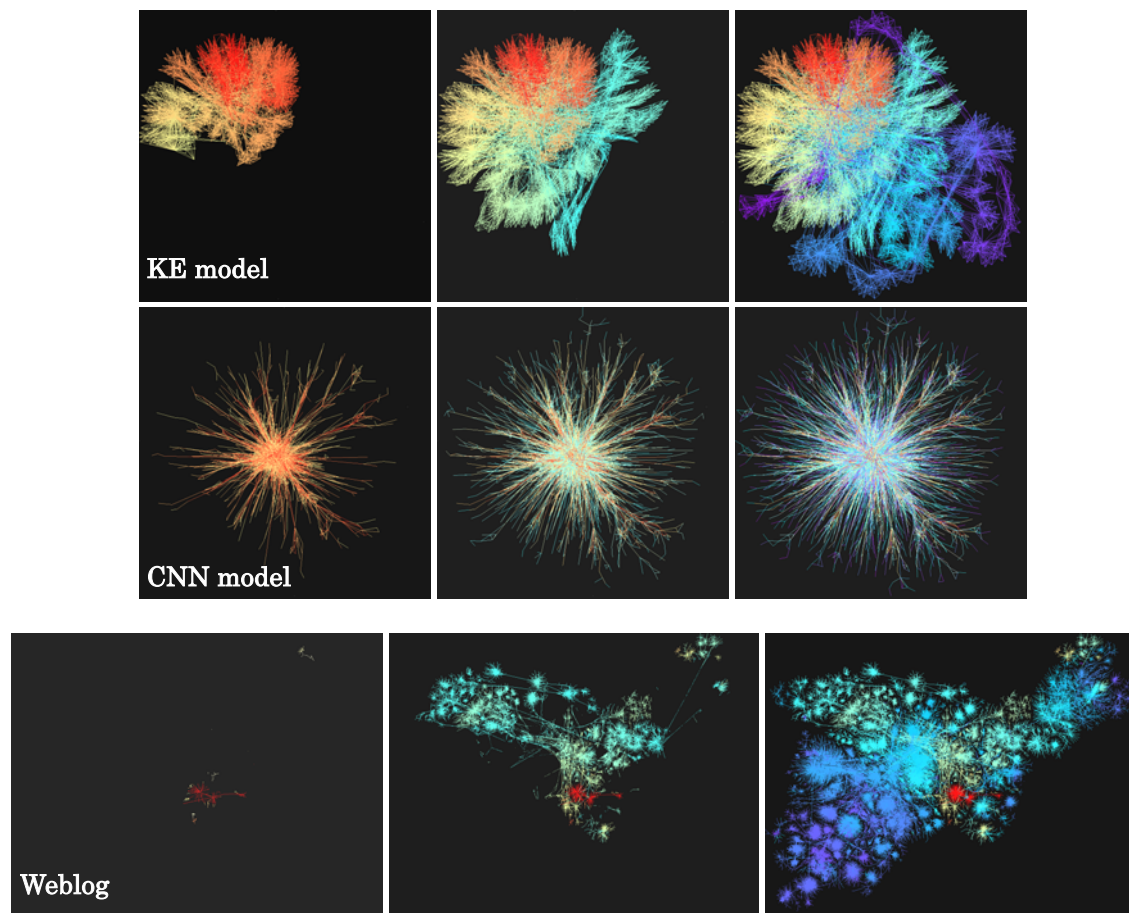


Fig. 3. (Continued).

5. Results and Discussion

We have tried to elicit several patterns on complex networks by visualizing structures of networks. Edge-centric drawing method for the different type of network features related to the static structure and growing behavior is proposed. It is shown that several patterns in complex networks can be extracted from colorizing the edges according to the hubs, the communities and the age in growing process.

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